

# A Modified Array Processing Technique Based on Kaiser Window for Concentric Circular Arrays

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**Abstract**— In this paper, an efficient beamforming technique is proposed where the Kaiser window is designed and applied to the uniform concentric circular arrays (UCCA) for sidelobe reduction. This technique is based on tapering the current amplitudes of the rings in the array, where all elements in an individual ring are weighted in amplitude by the same value. Based on establishing some mapping curves, this modified tapering window is optimized in its parameters to have the lowest possible sidelobe level that can be 45 dB below the main lobe level and these optimum weights are found to be function of the number of elements of the innermost ring and the number of rings in the array.

*Keywords*- Antenna Arrays, Beamforming, Circular Arrays

## I. INTRODUCTION

Array processing techniques have considerable interests recently and the array configuration affects its performance greatly. Among the different array configurations circular arrays have various applications including sonar, radar, and mobile communications [1-3]. It consists of a number of elements -usually omnidirectional- arranged on a circle [1] and can be used for beamforming in the azimuth plane for example at the base stations of the mobile radio communications system [3]. This array suffers from the higher sidelobe level (approximately 8 dB below the main lobe) especially when utilized in two-dimensional beamforming and the array is inefficient if utilized at angles near to the normal of the array. Therefore, one possible solution to reduce this higher sidelobe level is to use multiple concentric circular arrays (CCA) of different number of elements and radii. Uniform CCA (UCCA) is one of the most important configurations of the CCA [4] where the inter-element spacing in individual rings and the inter-ring spacing are kept almost half of the wavelength. The sidelobes in the UCCA will drop to about -17.5 dB especially at larger number of rings [4]. However, this sidelobe level still will be very high in some applications. Therefore, this paper is devoted for reducing the sidelobe level in UCCA using a Kaiser amplitude weighting applied to the rings of the array. This tapering window is optimized to provide the lowest possible side lobe levels. The paper is arranged as follows; in section 2 discusses the UCCA

geometry and section 3 introduces the proposed tapering window. In section 4, the Kaiser UCCA beamformer is introduced and the performance of beam power pattern, sidelobe levels and beamwidth is discussed in section 5. Finally section 6 concludes the paper.

## II. GEOMETRY OF THE UCCA

Concentric circular antenna arrays has elements arranged in multiple concentric circular rings which differ in radius and number of elements as shown in Figure 1, where there are  $M$  concentric circular rings. The  $m^{th}$  ring has a radius  $r_m$  and number of elements  $N_m$  where  $m = 1, 2, \dots, M$ .

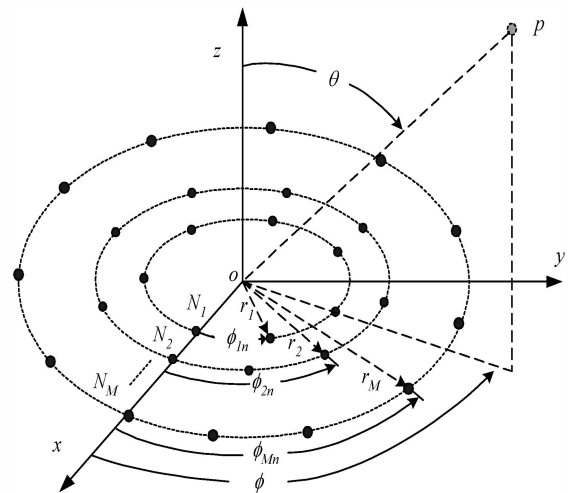


Figure 1: Concentric Circular Arrays (CCA)

In array processing, it is generally assumed that all elements in the array are omnidirectional sensors; therefore the power pattern can be defined if we know the weighting and steering matrices of the array. An expression for the array steering matrix has been deduced in [4] and is given by:

$$AS(\theta, \phi) =$$

$$\begin{bmatrix} e^{jkr_1 \sin \theta \cos(\phi - \phi_{11})} & e^{jkr_2 \sin \theta \cos(\phi - \phi_{21})} & \dots & e^{jkr_M \sin \theta \cos(\phi - \phi_{M1})} \\ e^{jkr_1 \sin \theta \cos(\phi - \phi_{12})} & e^{jkr_2 \sin \theta \cos(\phi - \phi_{22})} & \dots & e^{jkr_M \sin \theta \cos(\phi - \phi_{M2})} \\ \vdots & \vdots & \ddots & \vdots \\ e^{jkr_1 \sin \theta \cos(\phi - \phi_{1N_1})} & \dots & \dots & \dots \\ 0 & e^{jkr_2 \sin \theta \cos(\phi - \phi_{2N_2})} & \dots & \dots \\ 0 & 0 & \dots & \dots \\ 0 & 0 & \dots & e^{jkr_M \sin \theta \cos(\phi - \phi_{MN_{M-1}})} \\ 0 & 0 & \dots & e^{jkr_M \sin \theta \cos(\phi - \phi_{MN_M})} \end{bmatrix} \quad (1)$$

where the azimuth angle of the  $mn^{\text{th}}$  element in the array is given by:

$$\phi_{mn} = \frac{2\pi n}{N_m}, \quad n = 1, 2, 3, \dots, N_m \quad (2)$$

Each column in the array steering matrix represents the corresponding ring steering vector which in general for the  $m^{\text{th}}$  ring is given by:

$$S_m(\theta, \phi) = [e^{jkr_m \sin \theta \cos(\phi - \phi_{m1})} \quad e^{jkr_m \sin \theta \cos(\phi - \phi_{m2})} \quad \dots \quad e^{jkr_m \sin \theta \cos(\phi - \phi_{mN_m})}]^T \quad (3)$$

Therefore, the array steering matrix may be rewritten as:

$$AS(\theta, \phi) = [S_1(\theta, \phi) \quad S_2(\theta, \phi) \quad \dots \quad S_m(\theta, \phi) \quad \dots \quad S_M(\theta, \phi)] \quad (4)$$

It is usually for the ring steering vectors to have different lengths as they have different number of elements, therefore we append each column with zeros for lower length vectors in the array steering matrix.

If the interelement spacing in any array exceeds half of the wavelength, the resulted radiation pattern will have grating lobes of higher levels. On the other hand, if this distance is smaller than half of the wavelength, the pattern will have a wider main lobe and the mutual coupling between elements will increase. Therefore, the uniform concentric circular array (UCCA) configuration that has almost half of the wavelength separating distance either between neighboring elements in a ring or between any two neighboring rings is needed to have a reasonable radiation pattern and this can be obtained if the number of elements is incremented by 6 elements [4] or:

$$N_{m+1} = N_m + 6 \quad (5)$$

This gives inter-ring separation distance of  $0.4775\lambda$  (which is the nearest possible value to half of the wavelength) which occurs only if for any ring the distance between any two neighboring elements is set half of the wavelength or:

$$\frac{2\pi r_m}{N_m} = 0.5\lambda \quad (6)$$

We can control the radiation pattern of the array by controlling the magnitudes and phases of the exciting currents, therefore the array factor or gain will be determined by the following equation

$$G(\theta, \phi) = \text{SUM} \{ W(\theta, \phi)^H AS(\theta, \phi) \} \quad (7)$$

where the *SUM* operator is the summation of the elements of the resulted matrix and  $W(\theta, \phi)$  is the weight matrix that controls the amplitudes and phases of the input currents. To have a delay-and-sum beamformer, we can form the main lobe in the direction  $(\theta_o, \phi_o)$  by setting the weight matrix to equal the array steering matrix at the same direction.

### III. BEAMFORMER FOR KAISER UCCA

Non-uniform amplitude weighting such as in tapered beamforming will improve the radiation pattern of any type of antenna array and results in lower sidelobe levels at the cost of larger beamwidth depending on the type of the utilized tapering window. In the case of UCCA, the tapering may be applied with an analogy to a linear one-dimensional array, where the number of elements in the linear array is twice the number of rings; therefore, it will be usually an even number. The outermost ring will take the weight value that corresponds to the outermost elements of the linear array and proceeding inwardly to the innermost ring that has the same weighting as the two innermost elements in the one-dimensional array. Therefore, the highest amplitude weighting will be at the innermost ring while the smallest amplitudes are for the outermost ring. Beamforming using tapered amplitude windows in this case can be considered as a subarrays processing technique as shown in Fig. 2 where all elements that correspond to a ring will have the same amplitudes and this will reduce greatly the processing burden and calculations that find each element-amplitude by other adaptive algorithms [5].

For a one-dimensional array we assume that the sidelobe level is below the mainlobe level by  $A_r$  dB, and then calculating the value of  $\beta_o$  where it is given by [6]:

$$\beta_o = \begin{cases} 0, & \text{for } A_r \leq 21 \\ 0.5842(A_r - 21)^{0.4} + 0.07886(A_r - 21), & \text{for } 21 < A_r \leq 50 \\ 0.1102(A_r - 8.7), & \text{for } A_r > 50 \end{cases} \quad (8)$$

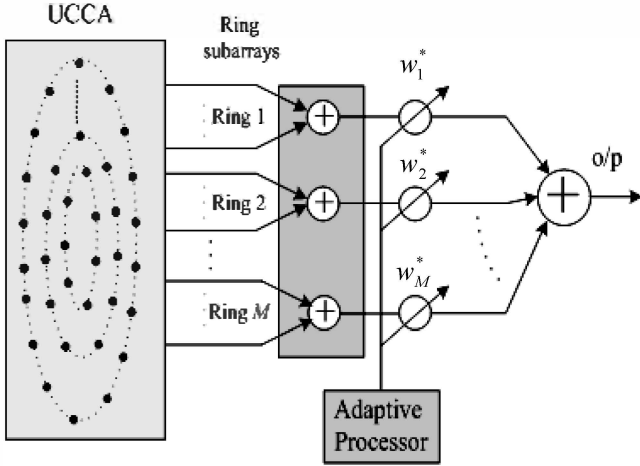


Figure 2: Tapered beamformer applied to ring subarrays

Therefore the amplitude coefficient of the  $m^{\text{th}}$  ring will be:

$$w_m = \frac{I_o \left[ \beta_o \sqrt{1 - \left( \frac{m-1}{M} \right)^2} \right]}{I_o(\beta_o)}, \quad \text{for } m = 1, 2, \dots, M \quad (9)$$

Where  $I_o$  is the zero<sup>th</sup>-order modified Bessel function of the first kind, which can be efficiently determined through its series expansion given by:

$$I_o(x) = 1 + \sum_{k=1}^{\infty} \left[ \frac{(x/2)^k}{k!} \right]^2 \quad (10)$$

The value of the sidelobe level  $A_r$  for the Kaiser linear one-dimensional array will not be the same as that resulting from the Kaiser UCCA, therefore, a mapping between the two levels is needed.

#### IV. RESULTS AND DISCUSSION

The normalized Kaiser window for a 10 ring UCCA with  $N_1 = 5$  and tapered at  $A_r = 80$  dB is shown in Fig. 3 and the corresponding normalized power pattern is depicted in Fig. 4 where the sidelobe level has dropped to less than -37.5 dB below the mainlobe level.

In general the Sidelobe level will be reduced as shown in Fig. 5 with increasing the values of  $A_r$ . Increasing  $A_r$  beyond 80 dB will not affect greatly the resulted sidelobe level and the subsequent curves will converge together.

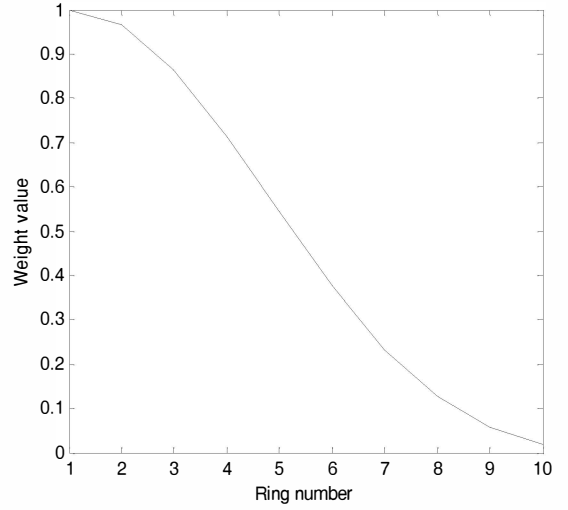


Figure 3: Kaiser window variation with the ring number in 10 rings UCCA at  $A_r = 80$  dB.

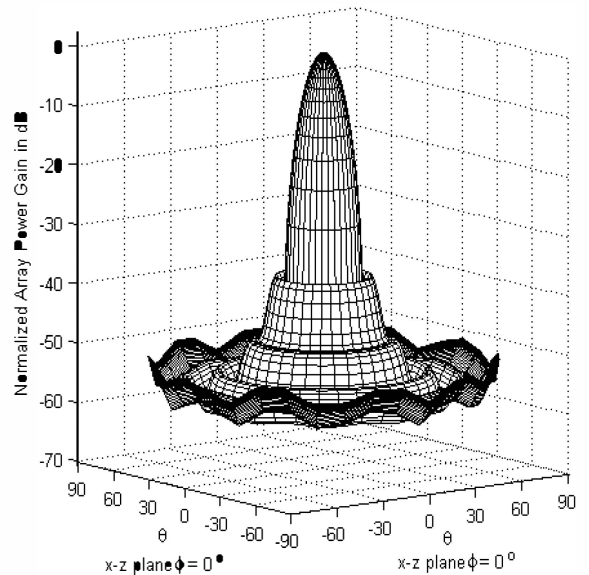


Figure 4: Normalized power pattern for Kaiser 10 rings UCCA at  $A_r = 80$  dB.

For  $A_r < 80$  dB, the sidelobe reduction by increasing the number of rings is insignificant as shown in the same figure at  $A_r = 40$  and 60 dB. On the other hand, the reduced sidelobe level increases the beamwidth as shown in Fig. 6.

Although at  $A_r > 80$  dB the sidelobe level will not decrease by considerable values, the beamwidth will increase;

therefore, it is not advantageous to increase  $A_r$  beyond 80 dB to effectively utilize the array.

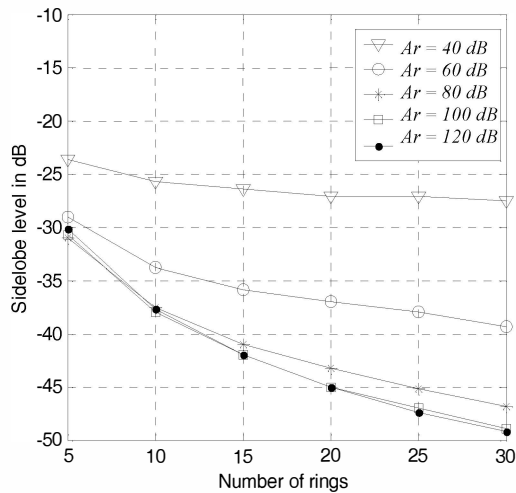


Figure 5: Sidelobe level versus number of rings for Kaiser-UCCA at different values of  $A_r$  and  $N_1 = 5$ .

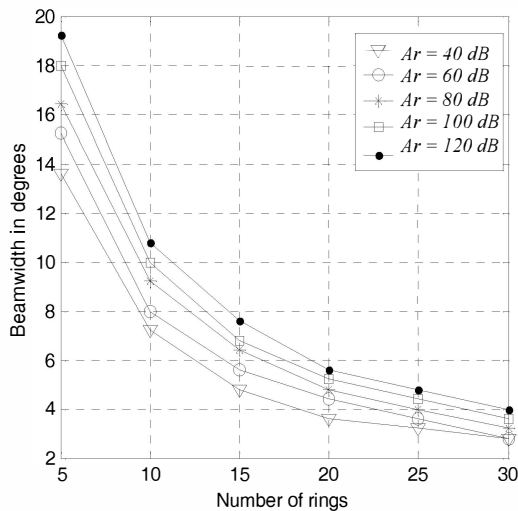


Figure 6: Beamwidth in degrees versus number of rings for Kaiser-UCCA at different values of  $A_r$  and  $N_1 = 5$ .

## V. CONCLUSION

In this paper we have applied the Kaiser coefficient weighting for the concentric circular arrays (UCCA). This weighting method is well-known for sidelobe reduction in linear arrays but not defined for UCCA, therefore we have proposed a method to apply the Kaiser weights directly to the arrays elements by dividing the UCCA into subarrays each

constituted by an individual ring array in which the elements are amplitude weighted by the same coefficient. By this way, a set of mapping curves is achieved for designing the array with the required sidelobe level and the performance of the sidelobe level and beamwidth is described, where the sidelobe level will be reduced effectively by increasing the number subarrays at values of  $A_r$  below 80 dB while the beamwidth is becoming wider after this value without extra decrease in the sidelobe level.

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